An Example Inductive Proof for MCS-236

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Theorem 1 *For any natural number n,*

$$\sum_{i=1}^{n}(2i-1) = n^2.$$

Proof. [induction] So long as the sum has at least one term (that is, $n > 0$), we can split the last term from the rest of the sum, writing

$$\sum_{i=1}^{n}(2i-1) = \sum_{i=1}^{n-1}(2i-1) + (2n-1). \tag{1}$$

Because $n-1$ is smaller than n and remains nonnegative, we can inductively assume that

$$\sum_{i=1}^{n-1}(2i-1) = (n-1)^2. \tag{2}$$

Substituting Equation 2 into Equation 1 and doing the algebra, we finish the inductive case as follows:

$$\begin{aligned}\sum_{i=1}^{n}(2i-1) &= (n-1)^2 + (2n-1) \\ &= n^2 - 2n + 1 + (2n-1) \\ &= n^2.\end{aligned}$$

Because this reasoning relied upon the sum having at least one term, we are left to consider $n = 0$ as a base case. When the sum is empty, its value is automatically 0, so we can confirm that

$$\sum_{i=1}^{0}(2i-1) = 0 = 0^2.$$

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