Work from 2011-10-14

MCS-236 class

October 14, 2011

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Theorem 1 *In an acyclic graph with m edges, n vertices, and k components, $k = n - m$.*

Proof. Each component is a tree, so in each component, there is one more vertex than edge. Summing all of the components, $k = n - m$. ■

We could also prove this in another, more elementary way. (Note that this still could use polishing.)

Proof. We can proceed by induction on m. If $m = 0$, then each vertex is a component and $k = n - 0$. Assume, then, that with $m - 1$ edges there are $n - (m - 1)$ components, that is, $n - m + 1$ components. Adding another edge brings the total edges to m and combines two components. To see that it combines two components, consider the alternative possibility that the new edge, sv, is within one of the components. Because that component is connected, it already had an $s - v$ path. Thus, adding the edge sv would complete a cycle, which is disallowed by the premise that the graph is acyclic. ■